Abstract—Orthogonal Frequency Division Multiplexing communication system is now starting to be widely used because of the high-speed data transfer. OFDM data transfer speeds can reach 100 Mbps. However, high-speed OFDM system transmits data makes it susceptible to fading and noise generated by the channel. Fading and noise can result in errors in the transmission of bits. Therefore, we need a technique that can reduce the error that occurred. One technique widely used is the estimated channel. Channel estimation is useful to reduce the changes that occur when the transmitted bits. In this thesis, will be explained one of the least square method of channel estimation with pilot symbol receipts. This estimator will estimate the channel containing the Rayleigh fading and AWGN to the receiver moving at certain speeds.

Key Words – channel, estimation, OFDM, fading, pilot,

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) are widely applied in mobile communication technology today because of the high transmission speed and bandwidth usage effectiveness when compared to its predecessor generation. This is why the OFDM technology has been proposed for broadband wireless access standards such as IEEE 802.16 (WiMAX) and as a core technique for the Long Term Evolution (LTE) fourth-generation wireless mobile communications [1]. But besides that, fading and noise is still a major problem in mobile communications because it can lead to errors and decrease the signal quality. Because, the higher data transfer rates the higher the required signal quality. Therefore, many, many ways used to overcome fading and noise in mobile communications. One way to do this is to estimate the channel[2]. Estimation channel using a variety of random distributions used to model the fading or noise occurs. In OFDM, channel estimation is needed, especially in fast fading channel, ie when the channel impulse response varies very rapidly with time.

In this journal will be explained about the system model channel estimation with pilot-assisted method using least squares estimation and simulation will be performed using a MATLAB program to find the performance of the channel estimation. Pilot-assisted channel estimation using pilot symbols inserted in the OFDM symbol. Pilot symbol is a signal that has previously been known that the effects of changes in the channel signal can be easily predicted. This technique is widely applied because of its ease in implementation and accurate in predicting the damage [3].

II. CHANNEL ESTIMATION

Channel estimation is a technique used in the transmission that aims to predict or estimate the channel impulse response (CIR) or the impulse response of a channel to the signal sent. Effect changes to the sent signal generated by the channel estimation must be done so that the detection signal becomes more accurate information.

In general, channel estimation can be grouped into three types, the pilot assisted channel estimation, blind channel estimation, and decision directed channel estimation. Pilot assisted channel estimation works by sending a pilot symbol
which is a sequence of bits that we have previously seen, along with the information to be sent. The pilot symbol is used to determine the pattern changes. Furthermore, with the pattern of these changes we can know the impulse response of channels. Hereinafter, with the interpolation method then information signal before passing through channel can we expect, so the error that occurred can be minimized. Unlike the pilot assisted channel estimation, blind channel estimation on the pilot symbols are not used to estimate the channel. Because we do not have to allocate specific bandwidth to transmit the pilot symbols, become more efficient use of bandwidth when compared with the the pilot assisted channel estimation techniques. Although the accuracy of estimating, the pilot assisted channel estimation techniques are still superior. Furthermore, the final estimation technique is the decision directed channel estimation. The basis of this technique is to use channel estimates obtained from the previous OFDM symbol channel estimation. Furthermore, the new estimate is obtained, is used to estimate the next. This technique is superior in estimating the bandwidth but in estimating is not better when compared with the pilot assisted channel estimation.

III. SYSTEM DECRIPTION

A. Pilot Symbol

As mentioned earlier, the pilot symbol is a row of bits that have previously been known. Previous recipients have learned the value of pilot symbols to be transmitted by the transmitter. Pilot symbols are sent with paste on the OFDM signal block of information. And then along with the pilot symbol signal information sent to the recipient. In general, there are two basic preparation of the pilot symbols are:

a. Block-type pilot, by including a pilot symbol into all subcarrier within a certain time within a specific time period. In addition to the previous recipient already knows the value of pilot symbols, the receiver also has to know when the pilot symbols transmitted simultaneously.

b. Comb type pilot, that is provided a special allocation of frequencies used to transmit pilot symbols every time. The sender determines the subcarrier which has previously been used to transmit pilot symbols.

![Figure 1. Two Types of Pilot Symbol Arrangement](image)

The arrangement of the pilot symbol on the type of block, the pilot symbols transmitted simultaneously in all the subcarriers at specified intervals. Therefore, the estimate used in the data symbol is the same for a finite time to do an estimate for the next time interval.

For the arrangement of the pilot symbol on the type of comb, a total $N_p$ pilot symbol of the signal $X_p(m), m = 0,1,...,N_p - 1$ are uniformly inserted into the signal $X(k)$. That is, of the total $N$ subcarriers are divided into groups where one group $N_p$ consists of $L = N/N_p$ group, the first subcarrier is used to transmit pilot signals modulated on the $k$-th subcarrier OFDM can be written as

$$X(k) = X(ml + l)$$

$$X(k) = \begin{cases} 
X_p(m), & l = 0 \\
\text{data information}, & l = 1,2,...L - 1 
\end{cases}$$

In these simulations will be used the arrangement of comb type pilot symbol. With a known pilot symbol value is transmitted and then when passing through the channel, the receiver can compare the value of pilot symbols that have previously been identified by the pilot symbol is changed through the channel. Furthermore, with the a channel estimation algorithm technique we can get his channel impulse response.

In the channel estimation block type, the pilot symbols transmitted by all subcarrier periodically. And further channel estimation is also performed regularly at the pilot symbol is sent. Therefore, these estimates are very suitable for frequency selective fading channel where there is need for different estimates on each individual subcarrier. Type channel estimation is also very appropriate when applied to slow fading channel with channel characteristics that have relatively fixed because of changes in the arrangement of block type pilot symbol estimation done at the interval at which pilot symbols sent.
While in non-pilot symbol estimation only followed by the pilot before.

On the other hand, the arrangement of comb type pilot symbol for pilot symbols are inserted at intervals of a fixed subcarrier at all times then this technique is more suited to fast channel fading. Impulse response at Fast fading channel varies very rapidly in each OFDM symbol, therefore the estimate needs to be done every time. This makes the comb type channel is more robust against fast fading. This technique is also suitable on flat fading channel where each frequency component of the signal fading experienced a relatively fixed magnitude. CIR value for subcarrier non-pilot (which contains the data) is estimated using the approach with the interpolation techniques.

Therefore, in OFDM systems where the channel is considered as flat fading or fast fading channel, preparation techniques comb type pilot symbol of a very good thing to do. At the end of this thesis, will be simulated OFDM system using comb-type pilot assisted channel estimation.

B. Least Square Channel Estimation

In general, there are many methods used in estimating channel. The number of methods is based on the reduction of error that occurred by comparing the pilot symbols are initially sent and received. Besides the method of least squares, method of channel estimation is widely used is the minimum mean square (MMSE), best linear unbiased estimator (BLUE), and adaptive boosting (AdaBoost) [4]. However, least square channel estimation was chosen because it is easier and very simple to apply. The difference technique is based on an algorithm taking CIR value of the comparison of known pilot symbol.

In OFDM systems, transmitters modulate a series of bits into symbols PSK / QAM, performed IFFT operation on the symbol to turn it into a signal in time domain, and further sent through the channel. Received signal is usually distorted by the channel characteristics. To repair bit sent, the effects of channel estimation should be expected or done. The equation of the received signal to the channel impulse response can be written into the equation

\[ Y = XH + W \]  \hspace{1cm} (3)

where \( Y \) is the received signal, \( H \) is the impulse response of channel, \( W \) is the noise, and \( X \) is the signal sent and each is written into the

\[ Y = [y[0], y[1], ..., y[N - 1]]^T \]  \hspace{1cm} (4)
\[ H = [h[0], h[1], ..., h[n - 1]]^T \]  \hspace{1cm} (5)
\[ W = [w[0], w[1], ..., w[N - 1]]^T \]  \hspace{1cm} (6)
\[ X = \begin{bmatrix} x[0] & 0 & ... & 0 \\ 0 & x[1] & ... & : \\ : & : & : & : \\ 0 & ... & 0 & x[N - 1] \end{bmatrix} \]  \hspace{1cm} (7)

\( X \) is written in the form of a diagonal matrix since we assume all orthogonal subcarrier.
The basic principle of Least Square Channel Estimation is by minimizing the error by using the method of Least Square Approach [6]. If \( x[n] \) is the signal sent and received signal after passing through channel, then error \( e[n] \) that occur can be formulated into the equation

\[
e[n] = y[n] - x[n]
\]

and least square error \( J(\theta) \) that is the square value of \( e[n] \) is

\[
J(\theta) = \sum_{n=0}^{N-1} (x[n] - y[n])^2
\]

With the substitution \( x[n] \) with the \( Y_p \) and \( y[n] \) with the \( \hat{H}_p X_p \) in equation (9) is obtained

\[
J(\theta) = (Y_p - \hat{H}_p X_p)^H (Y_p - \hat{H}_p X_p)
\]

Where \( (.)^H \) is the conjugate transpose operation. Minimum value of \( J(\theta) \) is achieved when \( \frac{\partial J(\theta)}{\partial \hat{H}} = 0 \), so that the obtained equation[5]

\[
\frac{\partial}{\partial \hat{H}} J(\theta) = -X_p^H Y_p + X_p^H X H = 0
\]

\[
H_{p,ls} = (X_p^H X_p)^{-1} X_p^H Y_p
\]

\[
H_{p,ls} = X_p^{-1} Y_p
\]

where \( H_{p,ls} \) is the impulse response least squares channel estimation. So that the pilot signal estimation based on least squares criteria is given with

\[
\hat{H}_{p,ls} = [H_{p,ls}(0) H_{p,ls}(1) H_{p,ls}(N_p - 1)]^T = X_p^{-1} Y_p
\]

C. Interpolation

After an estimate done and the estimated channel impulse response least squares \( \hat{H}_{p,ls} \) the obtained, interpolation technique is then performed. Interpolation is used to obtain the estimated channel impulse response in all OFDM symbols are sent. Two channel estimates obtained from the adjacent pilot symbols used for channel estimation on the data between them. There are various types of such one-dimensional interpolation, linear interpolation, cubic spine-interpolation, low-pass interpolation, and second-order interpolation. What is meant by this is a one-dimensional, we can interpolate with the review of one dimension only, and can we consider the frequency or time dimension. However, that will be used in channel estimation is linear interpolation here.

When the pilot symbols are distributed in the OFDM block by using structures such as comb-type pilots, interpolation carried out to obtain channel impulse response on the overall structure of the data subcarrier. By using interpolation, the estimated channel at the \( k \)-th subcarrier containing the information data in which \( mL < k < (m + 1)L \) is given with

\[
\hat{H}(k) = \hat{H}_{p,ls}(mL + 1), \quad 0 < l < L
\]

\[
\hat{H}(k) = \left(1 - \frac{l}{L}\right)\hat{H}_{p,ls}(m) + \frac{l}{L}\hat{H}_{p,ls}(m + 1)
\]

Where \( L \) is the number of subcarrier groups in comb-type and \( \hat{H}_{p,ls} \) is the carrier impulse response estimation. The value of \( \hat{H}(k) \) of each subcarrier is inserted at the beginning of the equation

\[
\tilde{X} = Y / \hat{H}
\]

In order to obtain the estimated value of the signal being sent or \( \tilde{X}[k] \) for all subcarriers.

IV. PERFORMANCE PARAMETERS

A. BER

BER or bit-error-rate is the ratio of the number of error bits or bits having errors with the all bits sent in the transmission of signals through a channel during a certain time interval. Bit error probability is the expected value of the BER. Therefore,
the BER can be determined by calculating the bit error probability.

Calculation of the BER on the AWGN channel is calculated by the integral Gaussian probability density function. Bit error rate for BPSK and QPSK written in equation

$$P_b = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right)$$

However, due to the channel impulse response $h$, the energy bit to noise ratio becomes $\frac{|h|^2 E_b}{N_0}$. So that the bit error probability becomes

$$P_{b|h} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{|h|^2 E_b}{N_0}} \right) = \frac{1}{2} \text{erfc}(\sqrt{\gamma})$$

where $\gamma = \frac{|h|^2 E_b}{N_0}$. $h$ is a random variable from the Rayleigh distribution, therefore $|h|^2$ is chi-square distribution with two degrees of freedom. Since $|h|^2$ is chi-square distribution, then $\gamma$ is also a chi-square distribution. Probability density function $\gamma$ that is

$$P(\gamma) = \frac{1}{E_b/N_0} e^{-\frac{\gamma}{E_b/N_0}}, \gamma \geq 0$$

By substituting equation (20) into equation (21) so that we get

$$P_b = \int_{0}^{\infty} \frac{1}{2} \text{erfc}(\sqrt{\gamma})P(\gamma)\, d\gamma$$

Equation (2.23) can be simplified into

$$P_b = \frac{1}{2} \left( 1 - \frac{(E_b/N_0)}{\sqrt{(E_b/N_0) + 1}} \right)$$

Because $\frac{E_b}{N_0} = SNR$, then we can change the above equation becomes

$$P_b = \frac{1}{2} \left( 1 - \sqrt{\frac{SNR}{SNR + \frac{B}{R}}} \right)$$

Where $B$ is the bandwidth and $R$ is the bit rate signal. In [7] described the Least Square Error Estimation with the equation

$$SNR_{LSE} = \frac{P_{LSE}}{N_{LSE}}$$

where $P_{LSE}$ is the signal power and noise power $N_{LSE}$ are respectively defined as

$$P_{LSE} = \frac{1}{K} \sum_{i=0}^{K-1} |y(i,j)\hat{h}(i,j)\alpha^*(i,j)|^2$$

and the noise power $N_{LSE}$ is defined as

$$N_{LSE} = \frac{1}{K} \sum_{i=0}^{K-1} |y(i,j)|^2 - P_{LSE}$$

where $\alpha$ is the amplitude of the QPSK modulated signal, $\hat{h}$ is the channel estimate, $y$ is the received signal and $i,j$ is the pilot carrier and the symbol index. Therefore, with the $SNR$ value substituted in equation (24) with the $SNR_{LSE}$ in equation (25) then we get the estimated signal BER.

B. Throughput

Throughput indicates the size of the number of data bits of information that is successfully delivered or the other in terms of number of packet symbols that are not experiencing an error in transmitting. Throughput is strongly influenced by the magnitude of the BER in the transmission of data. Throughput can be calculated with the equation,

$$Throughput = R(1 - PER)$$

where PER is packet error rate and $R$ is data rate transmission.

C. Channel capacity

Channel capacity is defined as the amount of information that can be transmitted through the channel. Channel capacity of a known CIR can be written using equation

$$C = \log_2 \left( 1 + \frac{SNR}{N} \right)$$

where $N$ is the noise power.
or can we write with the

\[ C = \log_2 \left\{ \det \left( 1 + \frac{P_{LSE}}{N_{LSE}} \cdot (\hat{H}_{is})^H \hat{H}_{is} \right) \right\} \] (29)

where \( \hat{H}_{is} \) is the channel response estimation and \( SNR_{LSE} \) is signal to noise estimation.

V. SIMULATION RESULTS AND ANALYSIS

Simulation least square channel estimation is made using the software MATLAB 7.8.0 (R2009a). This simulation aims to see the performance of least squares to estimate channel bit error rate, throughput, and channel capacity for the increased signal to noise ratio or SNR. For comparison, channel estimation is also carried out on various types of digital modulation, that is BPSK, QPSK, 16-QAM, and 64-QAM. The simulation was carried out on a stationary receiver and a moving receiver that generates a frequency shift or Doppler frequency of 100 Hz. The following are also included constellation each signal digital modulation on the value of SNR = 10 dB and SNR = 20 dB. This simulation has the following parameters:

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of subcarrier</td>
<td>256</td>
</tr>
<tr>
<td>Number of pilot symbol</td>
<td>32</td>
</tr>
<tr>
<td>Number of data subcarrier</td>
<td>224</td>
</tr>
<tr>
<td>Guard interval ratio</td>
<td>1/4</td>
</tr>
<tr>
<td>Channel Length</td>
<td>16</td>
</tr>
<tr>
<td>Modulation</td>
<td>BPSK, QPSK, 16-QAM, 64-QAM</td>
</tr>
<tr>
<td>Frequency Doppler</td>
<td>0 Hz dan 100 Hz</td>
</tr>
<tr>
<td>SNR</td>
<td>0 – 30 dB</td>
</tr>
<tr>
<td>IFFT size</td>
<td>256</td>
</tr>
</tbody>
</table>

In this simulation used 256 subcarriers where 32 subcarriers are used to transmit the pilot symbol so that the number of subcarriers is used to transmit data as much as 224. Or in another sense, one block of OFDM subcarriers is divided into 32 groups, each consisting of 8 subcarriers where the subcarriers beginning of each subcarriers group is used to transmit the pilot symbol. The system is maintained to avoid inter-symbol interference (ISI), hence the guard interval must be greater than the length of channel. Therefore, the channel length of 16 is added guard interval ratio of 1/4 or 64. In the system, do not over-sample so that the number of points equal to the number of IFFT subcarriers which is equal to 256.

A. BER

This section will explain the change of BER based the increase of SNR on a variety of modulation techniques with the stationary receivers and mobile receivers that produce the Doppler shift frequency of 100 Hz. For comparison, are included BER images system that do not use channel estimation techniques. Red straight line graph shows change of BPSK BER against SNR, while successively to green, blue, and black is a graph of BER of each modulation technique for QPSK, 16-QAM, and 64-QAM. BER calculations in these simulations are performed with the Monte Carlo technique that is sent bit by comparing the elements one by one with the elements of the received bits.

![Figure 3 BER vs SNR with the Doppler frequency = 0 Hz without any estimation technique](image-url)
In comparison, Figure 3 to Figure 4, the channel estimation can improve the BER value along with increasing SNR on BPSK and QPSK modulation techniques. However, for 16-QAM and 64-QAM, BER relative fixed although given the increase of SNR. At the SNR = 15 dB, the BER of 16-QAM and 64-QAM BER value does not decrease even after it was given increase of SNR. Accordingly as previously described, least square channel estimation cannot give a good effect on the BER at 16-QAM modulation technique and 64-QAM. Least square channel estimation works by finding the CIR obtained from the pilot symbols. In the high-order modulation techniques such as 16-QAM and 64-QAM, it seems like this is not good technique to be applied. The reason is, the higher order it will be more and more also the distribution of random numbers generated from the information which finally resulted in the possibility of CIR generated increasing irregularly. CIR irregularity is used extensively to estimate the value of bits that were previously sent. This makes the higher order modulation, channel estimation techniques to be applied poorly.

If we look at the picture, still there is a reduction the BER, but this did not mean. In contrast, in BPSK and QPSK modulation techniques, channel estimation managed to fix BER. Seen on the value of SNR = 9 dB, the BPSK and QPSK originally had BER value = 5.5x10^-2 and 44.6x10^-1, with the channel estimation technique BER at SNR equal to 4.5x10^-5 and 7.9x10^-3.

At any receiver moves, the channel estimation technique is shown in Figure 5, the BER is improved although still not as good as fixed receiver.

B. Throughput

Throughput of a system is very dependent on the packet error rate (PER) of the system. PER is also directly proportional to the BER that occur. The smaller the value of the BER will be smaller then the value of PER is also vice versa. Figure 6 shows the graphical relationship with the SNR on the throughput of modulation BPSK, QPSK, 16-QAM, and 64-QAM with Doppler frequency of = 0 Hz.
In the picture shown that at low SNR, which represents the channel conditions are poor, low-performance throughput from the system. In addition, the higher-order modulation, throughput is also higher. However, as discussed earlier, the simulation is estimated least square channel works effectively on BPSK and QPSK modulation, therefore the throughput graphs look better than 16-QAM and 64-QAM. Both modulation BER is very high, it is this which makes it smaller throughput of BPSK and QPSK although the 16-QAM and 64-QAM has a higher order.

Doppler frequency that occur can make the BER increases, if the graph in Figure 6 compared to the graph in Figure 7 looks throughput in Figure 7 is more sloping. It may be noted on the value of SNR = 6 dB. In BPSK and QPSK modulation throughput values in Figure 7 respectively about 210 and 310 whereas in Figure 6 for the same SNR value throughput 225 and 340.

C. Channel capacity

Figure 8 and Figure 9 shows the graphical capacity of the channel system with least square channel estimation at the receiver is stationary (Doppler frequency = 0 Hz) and the receiver are moving (frequency Doppler = 100 Hz). In the picture shown that the higher-order modulation of the channel capacity increases. This is because the higher-order modulation so the more bits are transmitted at the same time and make the canals increased capacity.

VI. CONCLUSION

From the simulation results and analysis has been done, then get some conclusions relating to the least square the channel estimation performance.
1. The channel estimation least square working optimally on BPSK and QPSK modulation techniques, but not when applied at higher modulation techniques such as 16-QAM and 64-QAM. In BPSK and QPSK modulation techniques, the channel estimation managed to fix the value of BER. Seen on the value of SNR = 9 dB, the BPSK and QPSK originally had BER value = 5.5x10^{-2} and 4.6x10^{-3}, with the channel estimation techniques SNR value at the same BER be 4.5x10^{-5} and 7.9x10^{-3}. While the 16-QAM and 64-QAM is relatively fixed.
2. In a moving receiver, the channel estimation may work well despite an increase of the value of the BER when compared with a stationary receiver. BER for BPSK, QPSK, 16-QAM, and 64-QAM for mobile receiver that generate the Doppler frequency of 100 Hz is 1.7x10^{-3}, 1.1x10^{-1}, 4x10^{-1}, 44.5x10^{-1}.
3. The capacity of the channel at the moving receiver slightly larger than a fixed receiver because the channel response matrix in conditions that make moving the greater capacity of canals to be enlarged accordingly. Figure 4.9 is obtained from each modulation BPSK, QPSK, 16-QAM, and 64-QAM capacity of canals by 5.6, 7.6, 12.25, and 4.16 for the value of SNR = 30 dB. Value of the highest throughput on the system with least square the channel estimation is achieved by QPSK modulation technique for 448, BPSK, 224, and further 64-QAM, 222, and the last 16-QAM, 83. optimally the channel estimation in the QPSK and BPSK, which makes the BER her down and this also makes PER declined but because the throughput is proportional to the QPSK data rate therefore have greater throughput than BPSK. Similarly, 64-QAM and 16-QAM. Only, the resulting BER and PER in the channel estimation on the modulation technique is very bad.

4. Value of the highest throughput on the system with least square the channel estimation is achieved by QPSK modulation technique for 448, BPSK, 224, and further 64-QAM, 222, and the last 16-QAM, 83. optimally the channel estimation in the QPSK and BPSK, which makes the BER her down and this also makes PER declined but because the throughput is proportional to the QPSK data rate therefore have greater throughput than BPSK. Similarly, 64-QAM and 16-QAM. Only, the resulting BER and PER in the channel estimation on the modulation technique is very bad.

VII. REFERENCE